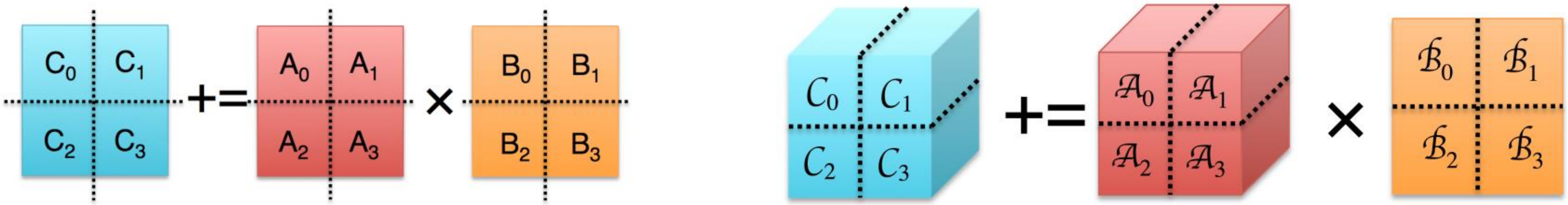


Strassen's Algorithm for Tensor Contraction

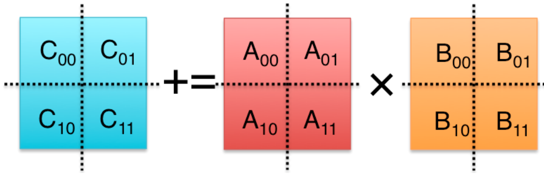
Jianyu Huang

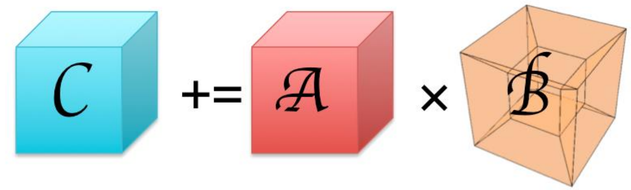
Joint work with Devin A. Matthews and Robert A. van de Geijn

The University of Texas at Austin

September 18-19, 2017
BLIS Retreat 2017

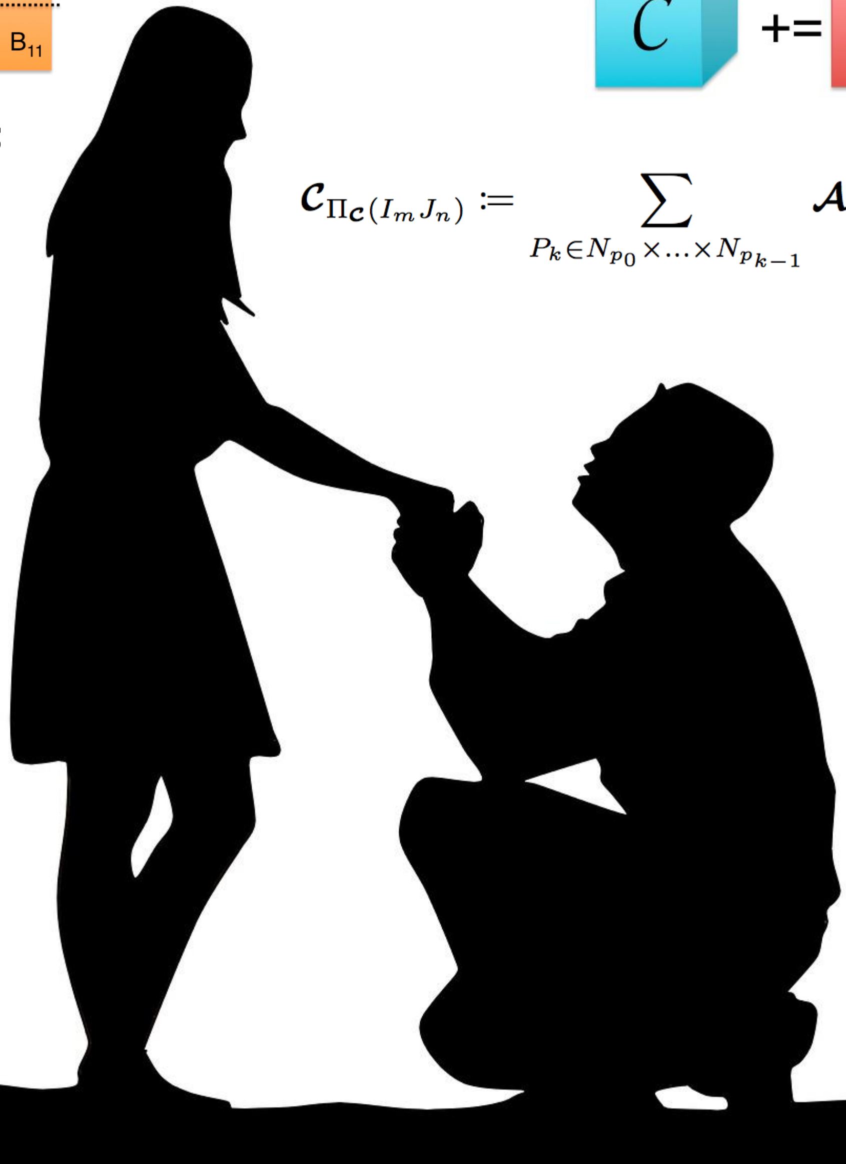
Marry Strassen with Tensor Contraction ?





$$\mathcal{C}_{\Pi_{\mathcal{C}}(I_m J_n)} := \sum_{P_k \in N_{p_0} \times \ldots \times N_{p_{k-1}}} \mathcal{A}_{\Pi_{\mathcal{A}}(I_m P_k)} \cdot \mathcal{B}_{\Pi_{\mathcal{B}}(P_k J_n)} + \mathcal{C}_{\Pi_{\mathcal{C}}(I_m J_n)}$$

$M_0 := (A_{00}+A_{11})(B_{00}+B_{11});$
$M_1 := (\mathrm{A}_{10}+\mathrm{A}_{11})\mathrm{B}_{00};$
$M_2 := A_{00}(B_{01}-B_{11});$
$M_3 := A_{11}(B_{10}-B_{00});$
$M_4 := (\mathrm{A}_{00}+\mathrm{A}_{01})\mathrm{B}_{11};$
$M_5 := (A_{10}-A_{00})(B_{00}+B_{01});$
$M_6 := (A_{01}-A_{11})(B_{10}+B_{11});$
$C_{00} \mathrel{+}= M_0 + M_3 - M_4 + M_6$
$C_{01} \mathrel{+}= M_2 + M_4$
$C_{10} \mathrel{+}= M_1 + M_3$
$C_{11} \mathrel{+}= M_0 - M_1 + M_2 + M_5$

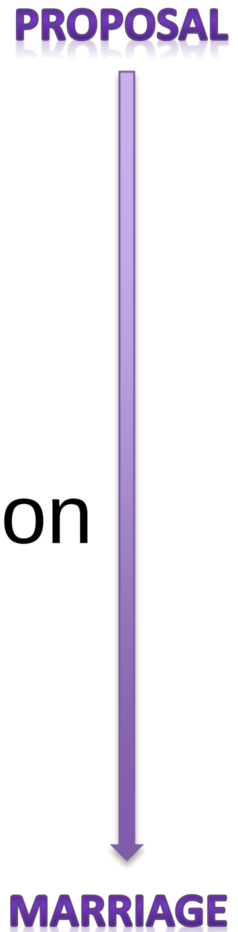


Practical Speedup?

$O(n^3) \rightarrow O(n^{2.8})$

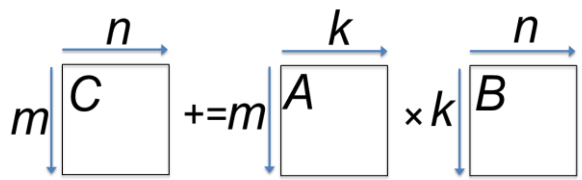
Outline

- Background
  - High-performance GEMM
  - High-performance Strassen
  - High-performance Tensor Contraction
- Strassen’s Algorithm for Tensor Contraction
- Performance Model
- Experiments
- Conclusion



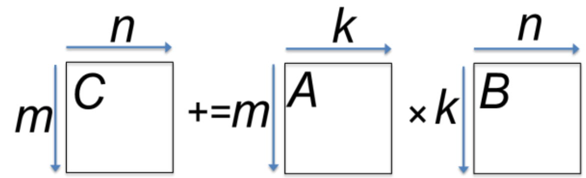
High-performance matrix multiplication (GEMM)

State-of-the-art GEMM in BLIS



- BLAS-like Library Instantiation Software (BLIS) is a portable framework for instantiating BLAS-like dense linear algebra libraries.
  - ❑Field Van Zee, and Robert van de Geijn. "BLIS: A Framework for Rapidly Instantiating BLAS Functionality." *ACM TOMS* 41.3 (2015): 14.
- BLIS provides a refactoring of GotoBLAS algorithm (best-known approach on CPU) to implement GEMM.
  - ❑Kazushige Goto, and Robert van de Geijn. "High-performance implementation of the level-3 BLAS." *ACM TOMS* 35.1 (2008): 4.
  - ❑Kazushige Goto, and Robert van de Geijn. "Anatomy of high-performance matrix multiplication." *ACM TOMS* 34.3 (2008): 12.
- GEMM implementation in BLIS has 6-layers of loops. The outer 5 loops are written in C. The inner-most loop (micro-kernel) is written in assembly for high performance.

GotoBLAS algorithm for GEMM in BLIS



Loop 5 for $j_C = 0 : n-1$ steps of n_C

$\mathcal{J}_C = j_C : j_C + n_C - 1$

Loop 4 for $p_C = 0 : k-1$ steps of k_C

$\mathcal{P}_C = p_C : p_C + k_C - 1$

$\boxed{B(\mathcal{P}_C, \mathcal{J}_C) \rightarrow \widetilde{B}_p}$

Loop 3 for $i_C = 0 : m-1$ steps of m_C

$\mathcal{I}_C = i_C : i_C + m_C - 1$

$\boxed{A(\mathcal{I}_C, \mathcal{P}_C) \rightarrow \widetilde{A}_i}$

// macro-kernel

Loop 2 for $j_r = 0 : n_C - 1$ steps of n_r

$\mathcal{J}_r = j_r : j_r + n_r - 1$

Loop 1 for $i_r = 0 : m_C - 1$ steps of m_r

$\mathcal{I}_r = i_r : i_r + m_r - 1$

//micro-kernel

Loop 0 for $p_r = 0 : p_C - 1$ steps of 1

$C_C(\mathcal{I}_r, \mathcal{J}_r) \mathrel{+}= \widetilde{A}_i(\mathcal{I}_r, p_r)\ \widetilde{B}_p(p_r, \mathcal{J}_r)$

endfor

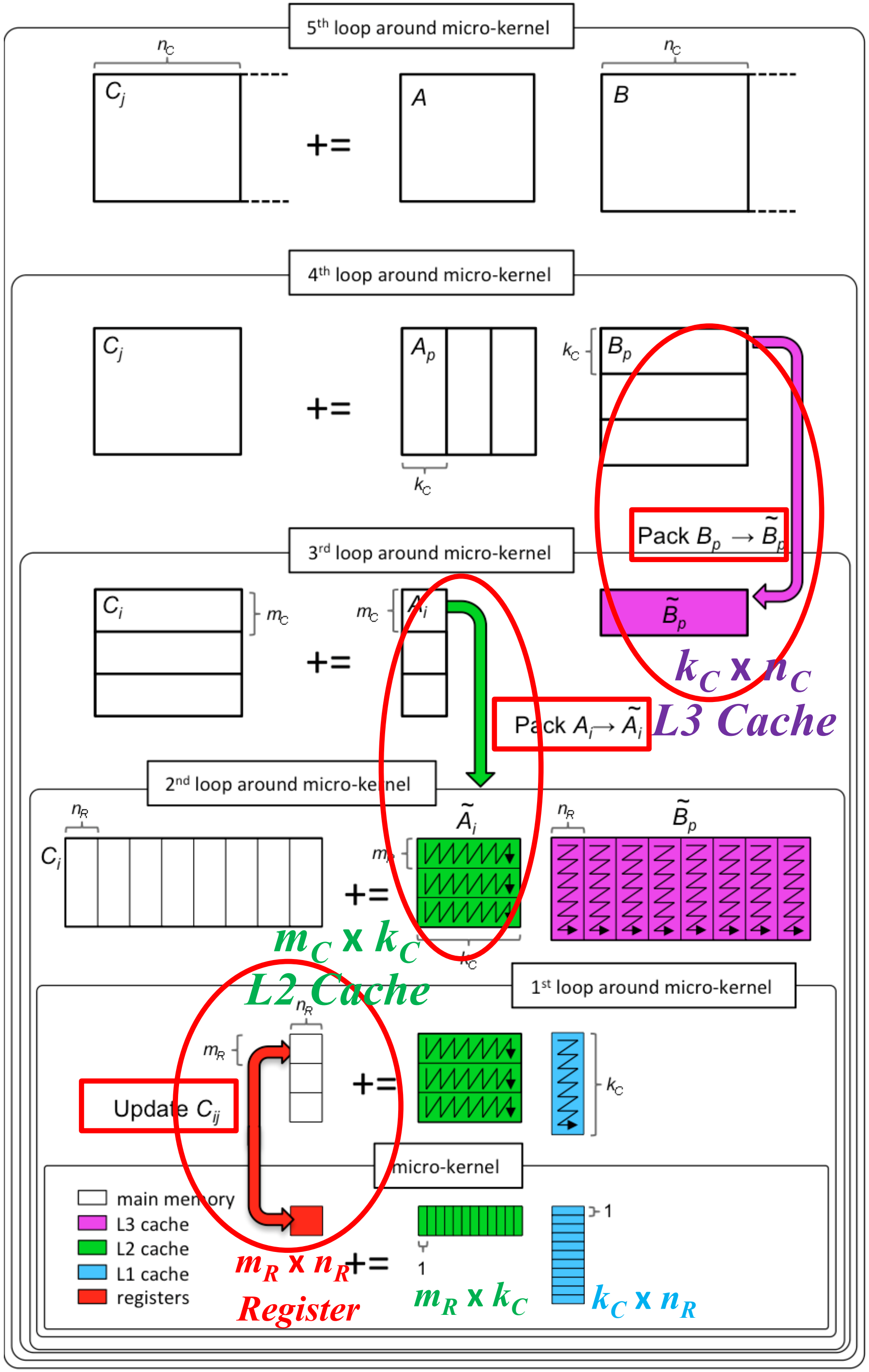
endfor

endfor

endfor

endfor

endfor



*Field G. Van Zee, and Tyler M. Smith. “Implementing high-performance complex matrix multiplication via the 3m and 4m methods.” In *ACM Transactions on Mathematical Software (TOMS), accepted.*

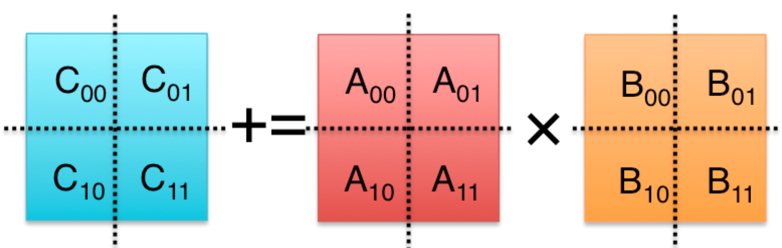
High-performance Strassen

*Jianyu Huang, Tyler Smith, Greg Henry, and Robert van de Geijn. "Strassen's Algorithm Reloaded." In *SC'16*.

Strassen's Algorithm Reloaded

$M_0 := (A_{00}+A_{11})(B_{00}+B_{11});$
$M_1 := (A_{10}+A_{11})B_{00};$
$M_2 := A_{00}(B_{01}-B_{11});$
$M_3 := A_{11}(B_{10}-B_{00});$
$M_4 := (A_{00}+A_{01})B_{11};$
$M_5 := (A_{10}-A_{00})(B_{00}+B_{01});$
$M_6 := (A_{01}-A_{11})(B_{10}+B_{11});$
$C_{00} \mathrel{+}= M_0 + M_3 - M_4 + M_6$
$C_{01} \mathrel{+}= M_2 + M_4$
$C_{10} \mathrel{+}= M_1 + M_3$
$C_{11} \mathrel{+}= M_0 - M_1 + M_2 + M_5$

$\boxed{M_0 := (A_{00}+A_{11})(B_{00}+B_{11}); \quad C_{00} \mathrel{+}= M_0;\ C_{11} \mathrel{+}= M_0;}$
$M_1 := (A_{10}+A_{11})B_{00}; \quad C_{10} \mathrel{+}= M_1;\ C_{11} \mathrel{-}= M_1;$
$M_2 := A_{00}(B_{01}-B_{11}); \quad C_{01} \mathrel{+}= M_2;\ C_{11} \mathrel{+}= M_2;$
$M_3 := A_{11}(B_{10}-B_{00}); \quad C_{00} \mathrel{+}= M_3;\ C_{10} \mathrel{+}= M_3;$
$M_4 := (A_{00}+A_{01})B_{11}; \quad C_{01} \mathrel{+}= M_4;\ C_{00} \mathrel{-}= M_4;$
$M_5 := (A_{10}-A_{00})(B_{00}+B_{01}); \quad C_{11} \mathrel{+}= M_5;$
$M_6 := (A_{01}-A_{11})(B_{10}+B_{11}); \quad C_{00} \mathrel{+}= M_6;$



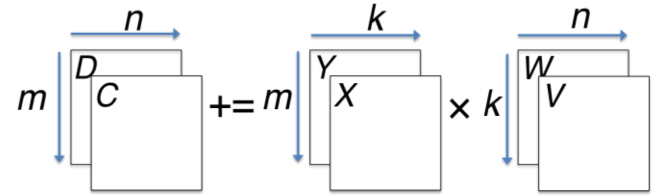
$\boxed{M := (X+Y)(V+W); \quad C \mathrel{+}= M; \quad D \mathrel{+}= M;}$

General operation for one-level Strassen:

$\boxed{\begin{array}{l} M := (X+\delta Y)(V+\varepsilon W); \quad C \mathrel{+}= \gamma_0 M;\ D \mathrel{+}= \gamma_1 M; \\ \gamma_0, \gamma_1, \delta, \varepsilon \in \{-1, 0, 1\}. \end{array}}$

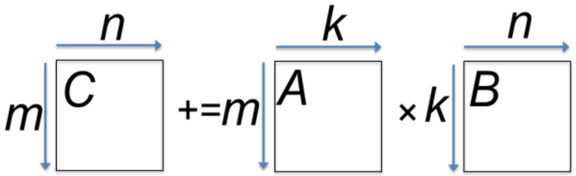
*__Jianyu Huang__, Tyler Smith, Greg Henry, and Robert van de Geijn. "Strassen's Algorithm Reloaded." In *SC'16*.

$$M := (X+Y)(V+W); \;\; C += M; \; D += M;$$

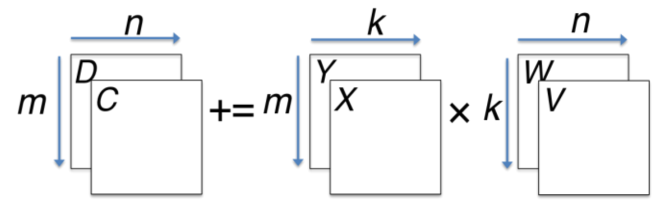


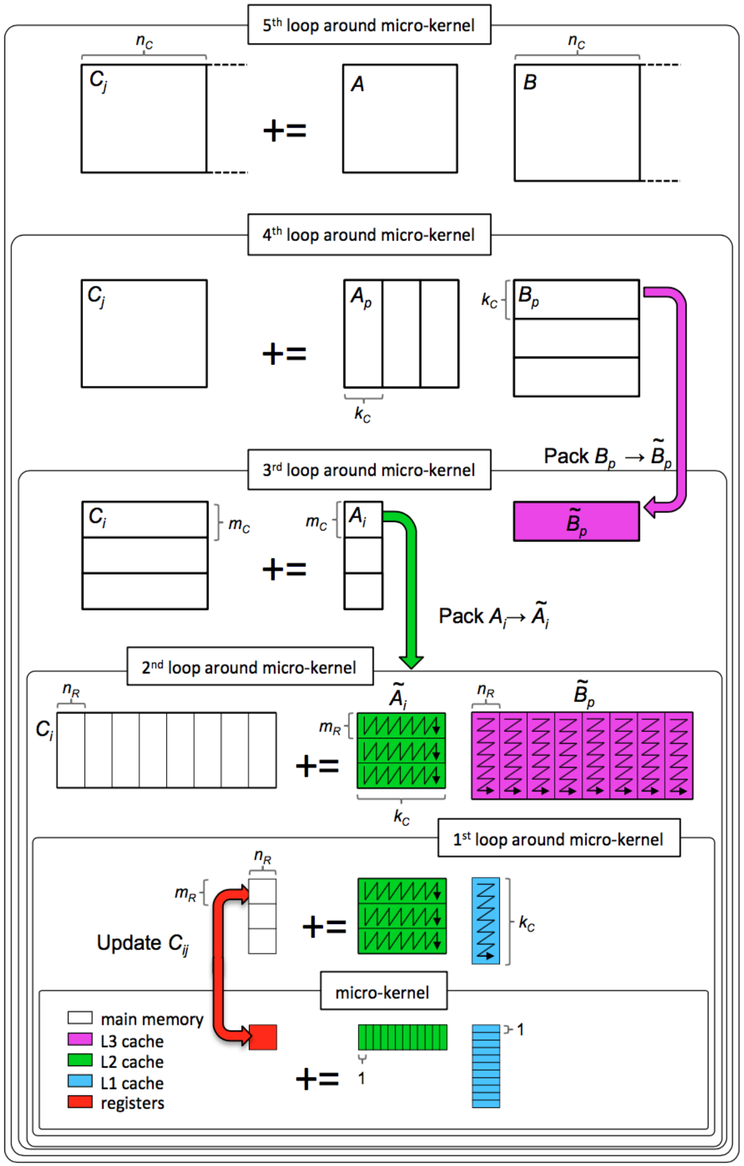
*__Jianyu Huang__, Tyler Smith, Greg Henry, and Robert van de Geijn. "Strassen's Algorithm Reloaded." In *SC'16*.

$C += AB;$

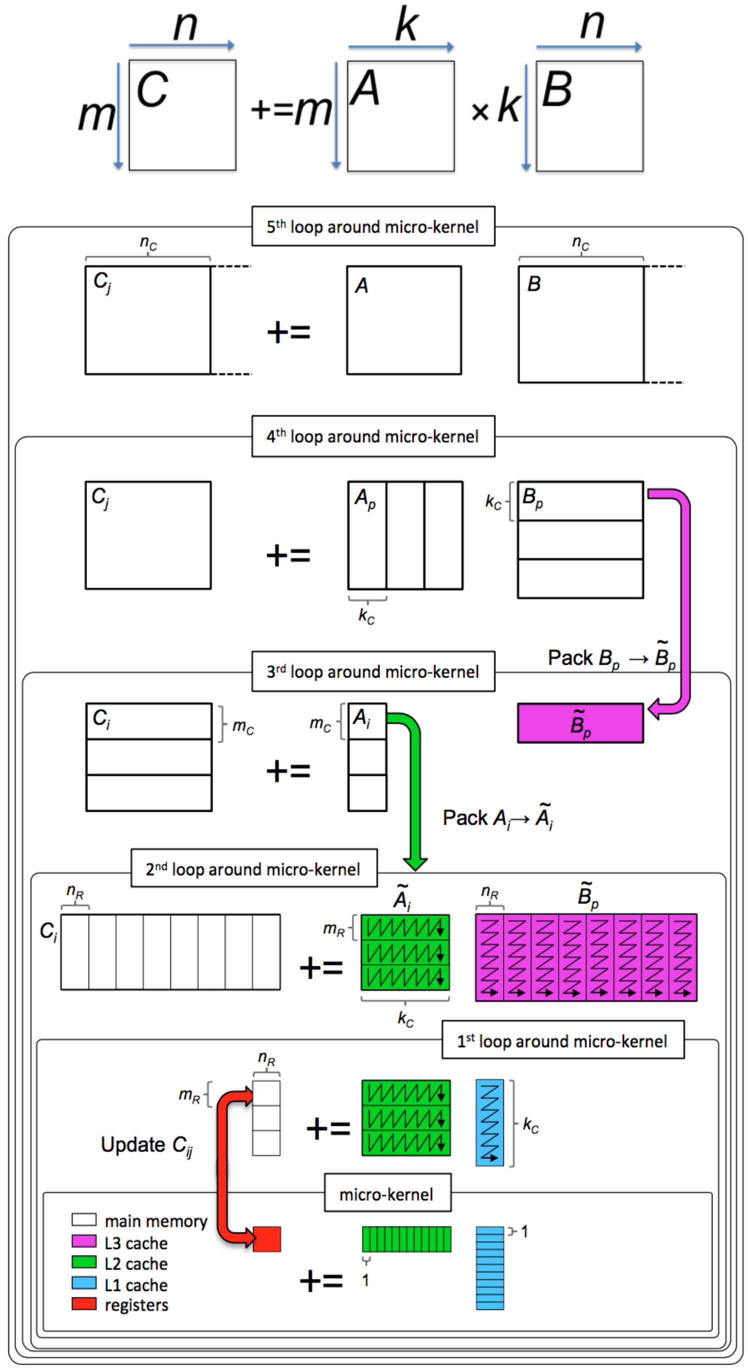


$M := (X+Y)(V+W); \; C += M; \; D += M;$

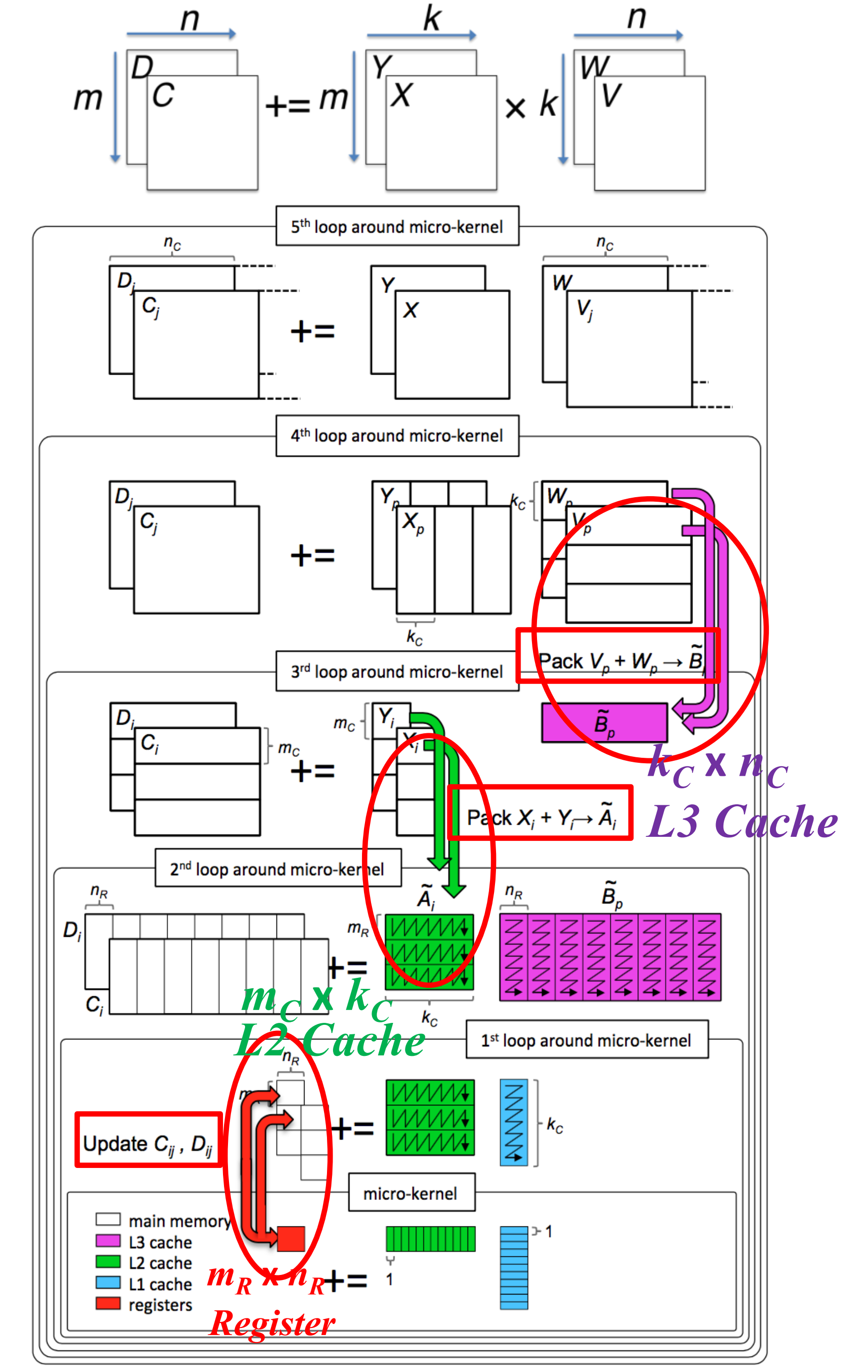




$C += AB;$



$M := (X + Y)(V + W); \; C += M; D += M;$

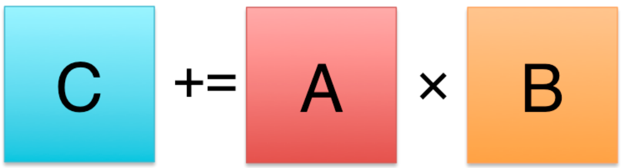


High-performance Tensor Contraction

Devin A. Matthews. “High-Performance Tensor Contraction without Transposition.” Accepted in *SISC*.

Matrix vs. Tensor

Matrix Multiplication

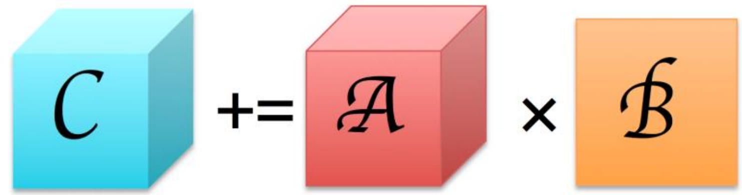


$$C += AB$$

$$C_{i,j} += \sum_k A_{i,k} B_{k,j}$$

BLAS/BLIS!

Tensor Contraction



$$\mathcal{C} += \mathcal{A}\mathcal{B}$$

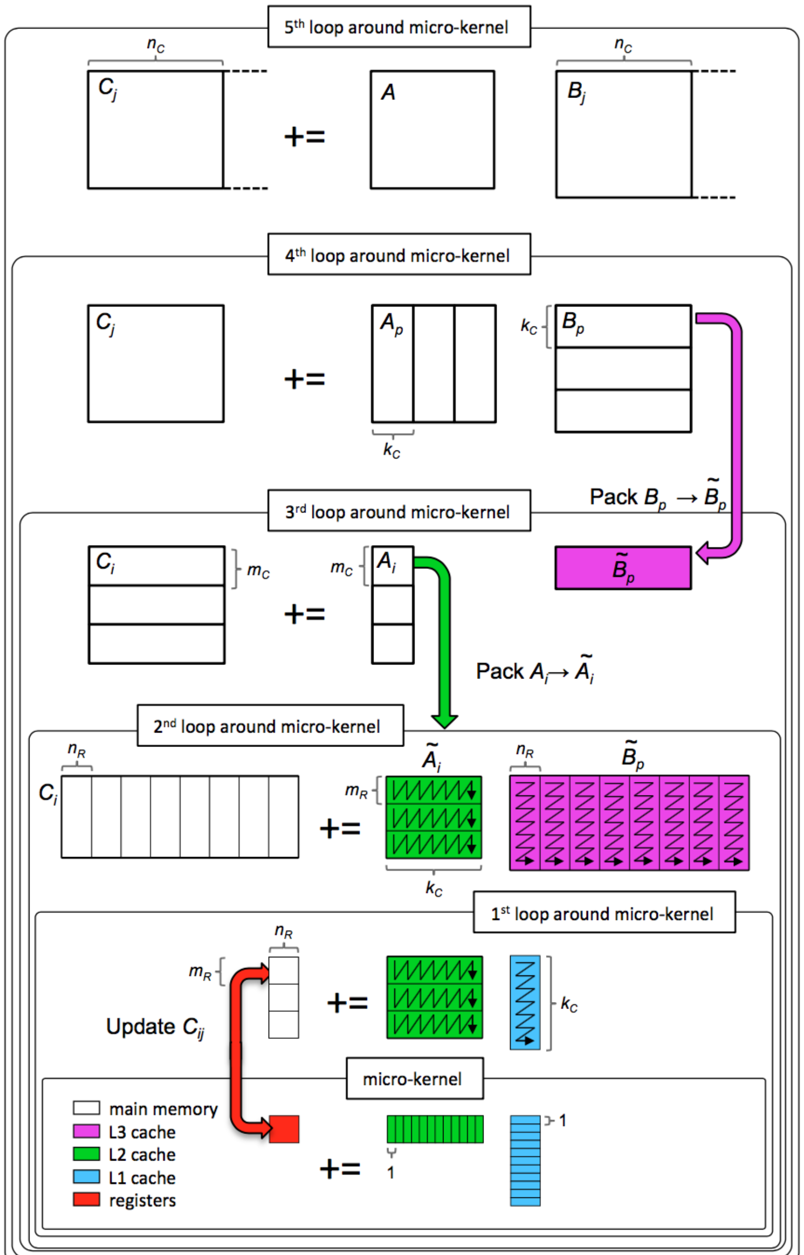
$$\mathcal{C}_{a,b,c} += \sum_d \mathcal{A}_{d,c,a} \mathcal{B}_{d,b}$$

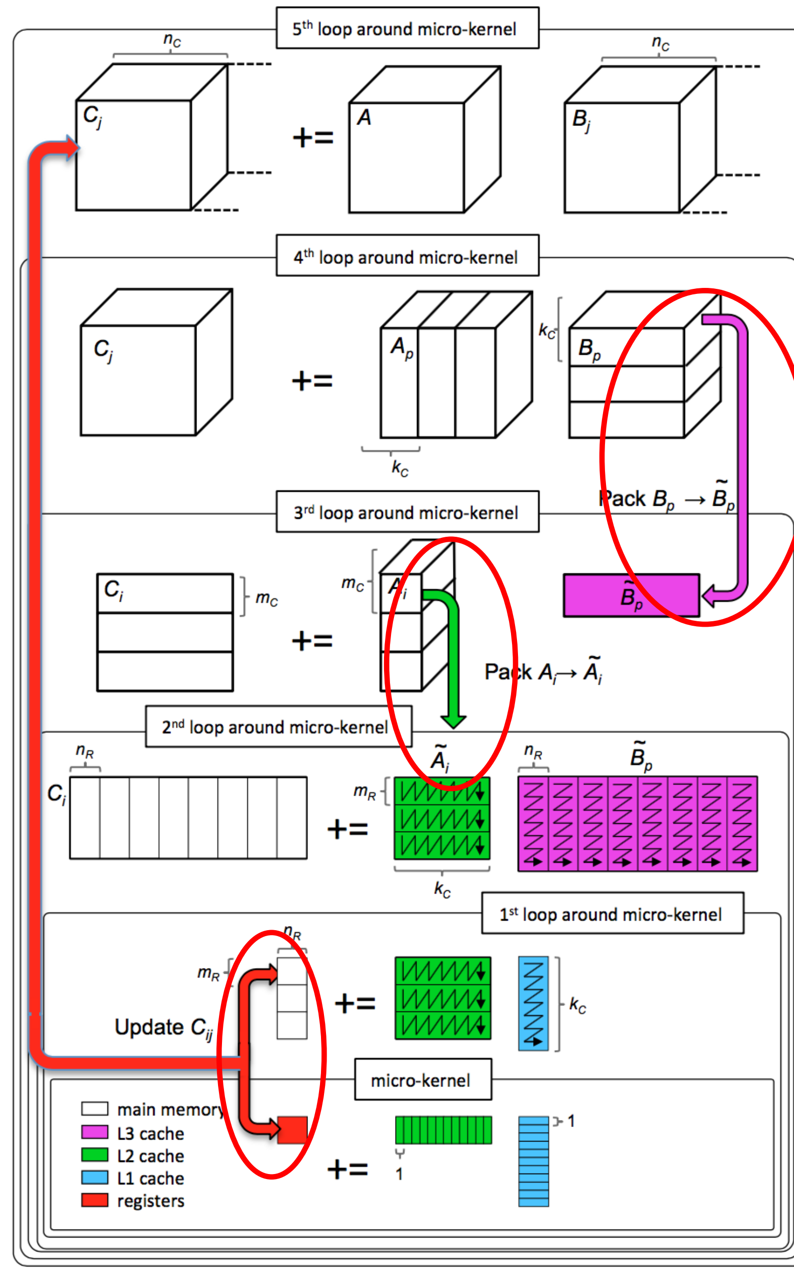
TBLIS!

Devin A. Matthews. “High-Performance Tensor Contraction without Transposition.” Accepted in *SISC*.

$$C := AB + C$$

$$\mathcal{C}_{\Pi_{\mathcal{C}}(I_m J_n)} := \sum_{P_k \in N_{p_0} \times \ldots \times N_{p_{k-1}}} \mathcal{A}_{\Pi_{\mathcal{A}}(I_m P_k)} \cdot \mathcal{B}_{\Pi_{\mathcal{B}}(P_k J_n)} + \mathcal{C}_{\Pi_{\mathcal{C}}(I_m J_n)}$$





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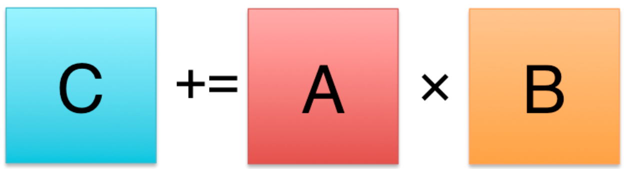
Matrix vs. Tensor

Matrix Multiplication

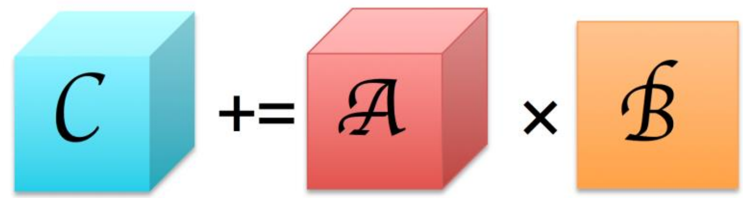


$$C \mathrel{+}= AB$$

$$C_{i,j} \mathrel{+}= \sum_k A_{i,k} B_{k,j}$$

BLAS/BLIS!

Tensor Contraction



$$\mathcal{C} \mathrel{+}= \mathcal{A}\mathcal{B}$$

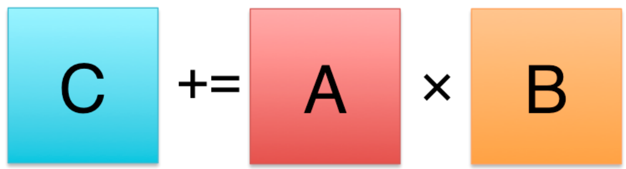
$$\mathcal{C}_{a,b,c} \mathrel{+}= \sum_d \mathcal{A}_{d,c,a} \mathcal{B}_{d,b}$$

TBLIS!

Devin A. Matthews. "High-Performance Tensor Contraction without Transposition." Accepted in *SISC*.

Matrix vs. Tensor

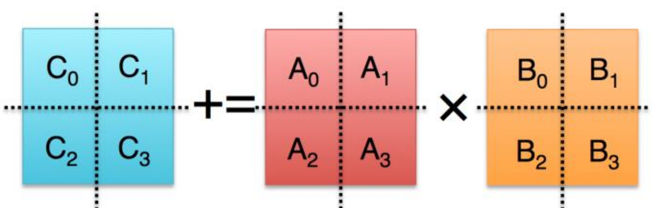
Matrix Multiplication



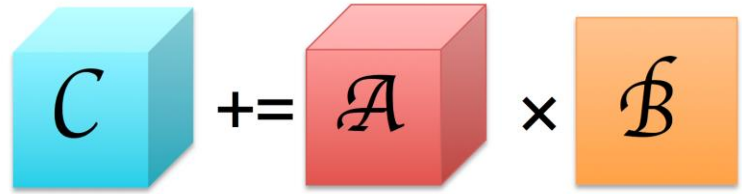
$$C += AB$$

$$C_{i,j} += \sum_k A_{i,k} B_{k,j}$$

BLAS/BLIS!



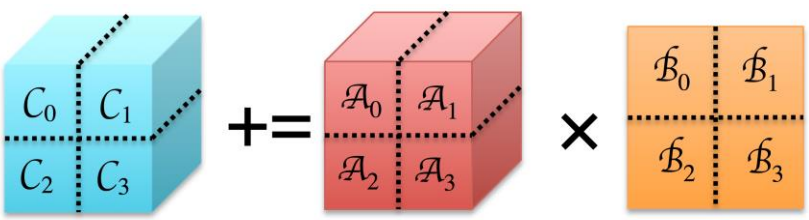
Tensor Contraction



$$\mathcal{C} += \mathcal{A}\mathcal{B}$$

$$\mathcal{C}_{a,b,c} += \sum_d \mathcal{A}_{d,c,a} \mathcal{B}_{d,b}$$

TBLIS!



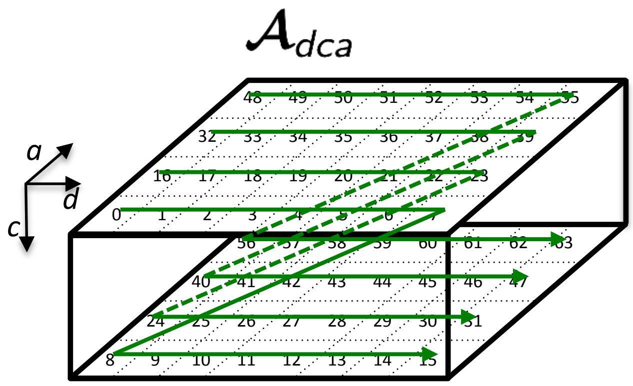
Devin A. Matthews. “High-Performance Tensor Contraction without Transposition.” Accepted in *SISC*.

Tensors As Matrices: Block-Scatter-Matrix View

- **Tensor**: $\mathcal{A}_{dca}$, 8x2x4

with $N_a = 4, N_c = 2, N_d = 8$

- "d" dimension is stride-1, other dimensions have increasing strides (8, 16).



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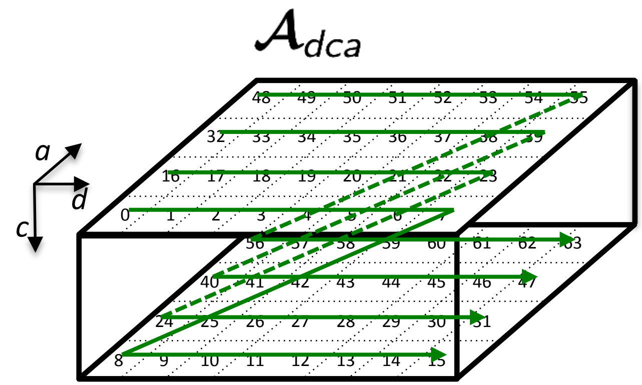
Tensors As Matrices: Block-Scatter-Matrix View



- **Tensor**: $\mathcal{A}_{dca}$, 8x2x4

with $N_a = 4, N_c = 2, N_d = 8$

- ❑ "d" dimension is stride-1, other dimensions have increasing strides (8, 16).

- **Matrix**: $A_{(ac)(d)}$, 8x8

with $N_{I_m} = N_a \cdot N_c = 8, N_{P_k} = N_d = 8$

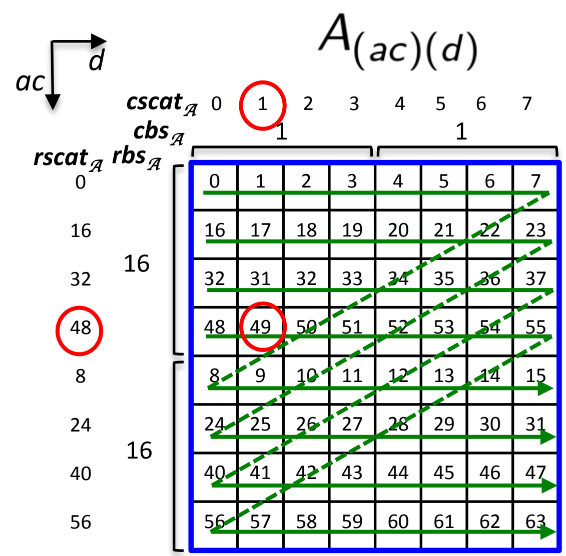
- ❑ Column "ac" dimension has stride of "c" (8x2=16).

Row "d" dimension has is stirde-1. (i.e. A is row-major.)

- ❑ $rscat_{\mathcal{A}}$, $cscat_{\mathcal{A}}$ store offset for each position in rows or columns: Scatter-Matrix Vector

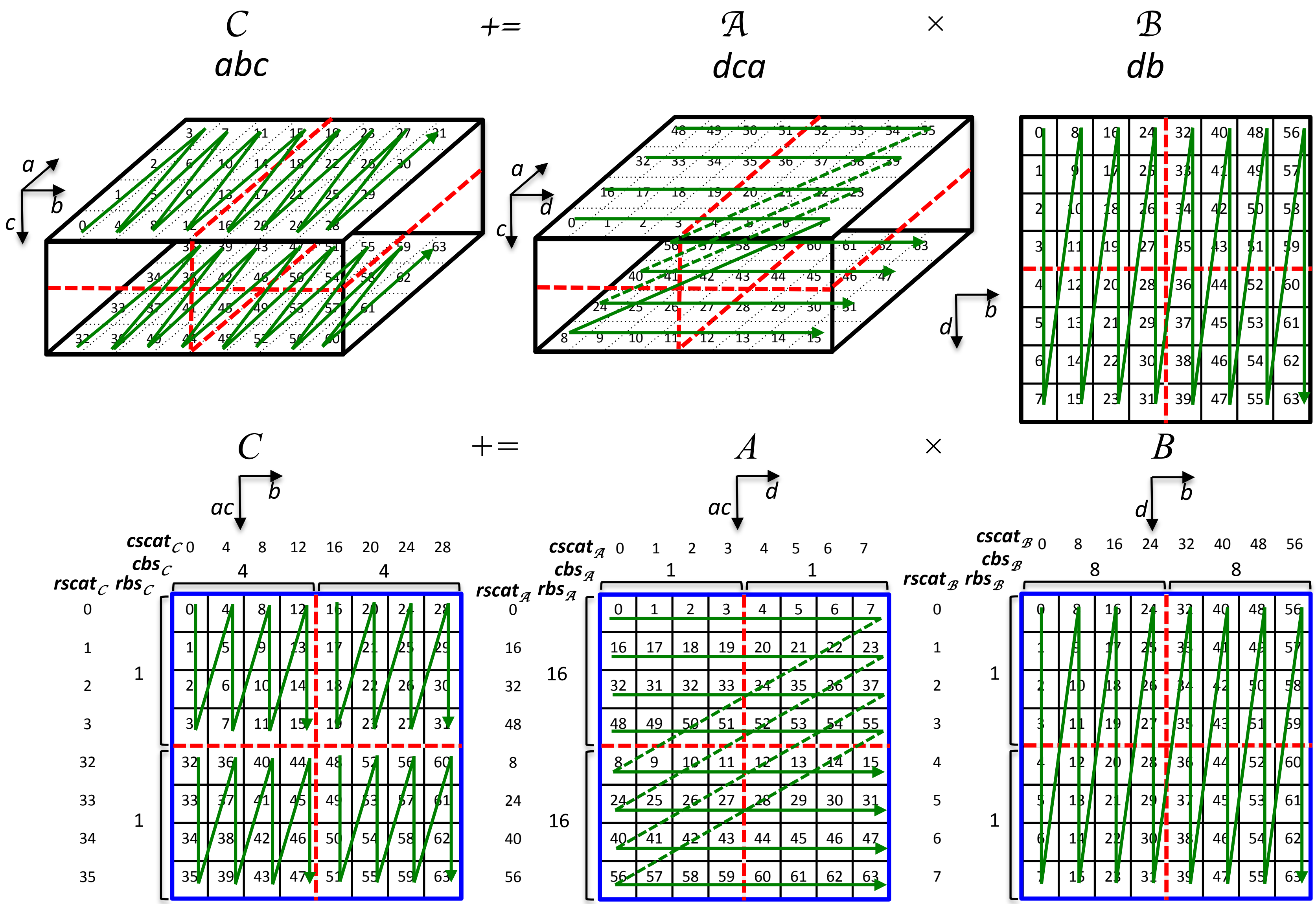
$$\mathrm{OFFSET}_{\mathcal{A}_{d,c,a}} = rscat_{A;(ac)} + cscat_{A;(d)}$$

- ❑ $rbs_{\mathcal{A}}$, $cbs_{\mathcal{A}}$ store stride for each block or zero for irregular blocks: Block-Scatter-Matrix Vector
  - vector load/store instructions for stride-1 index
  - vector gather/scatter instructions for stride-n index.



Devin A. Matthews. "High-Performance Tensor Contraction without Transposition." Accepted in *SISC*.

Strassen's Algorithm for Tensor Contraction



Jianyu Huang, Devin A. Matthews, and Robert A. van de Geijn. "Strassen's Algorithm for Tensor Contraction." arXiv:1704.03092 (2017).

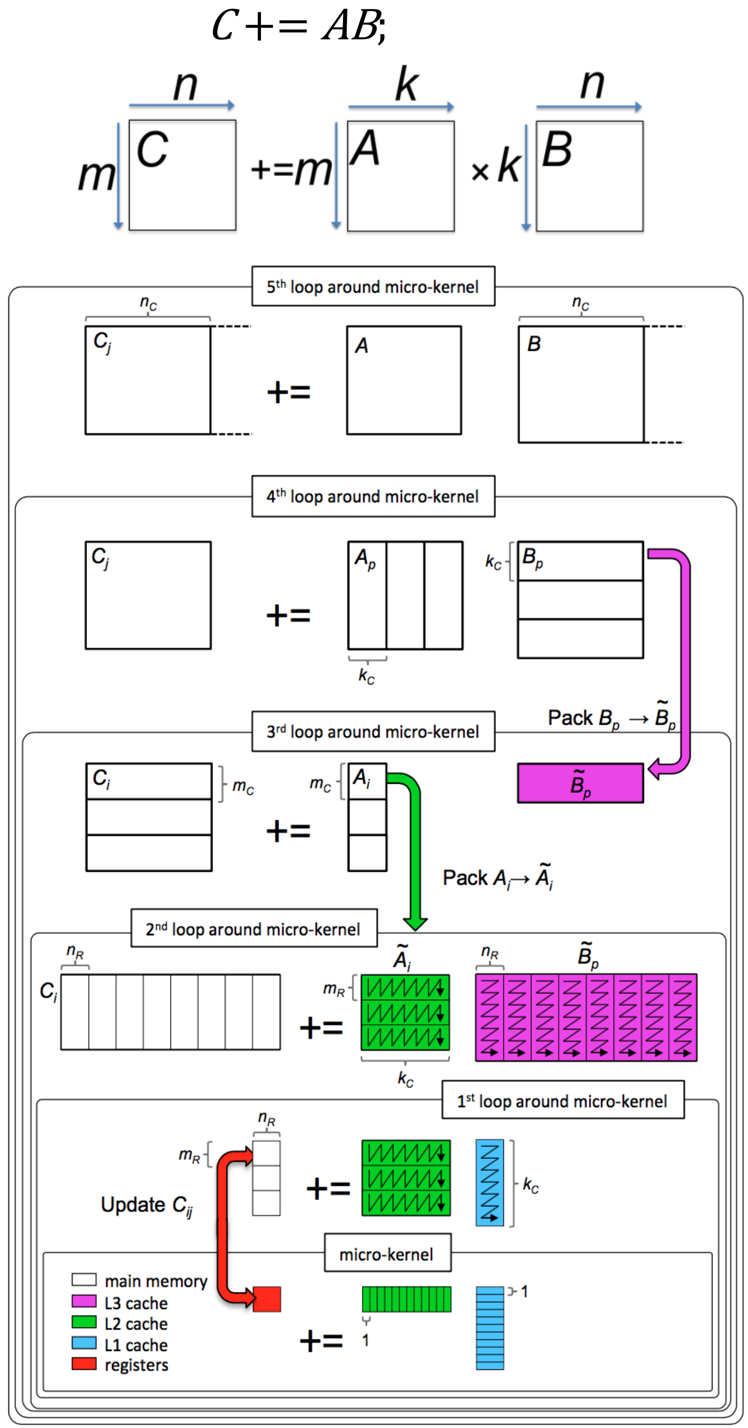
Modifications to GEMM

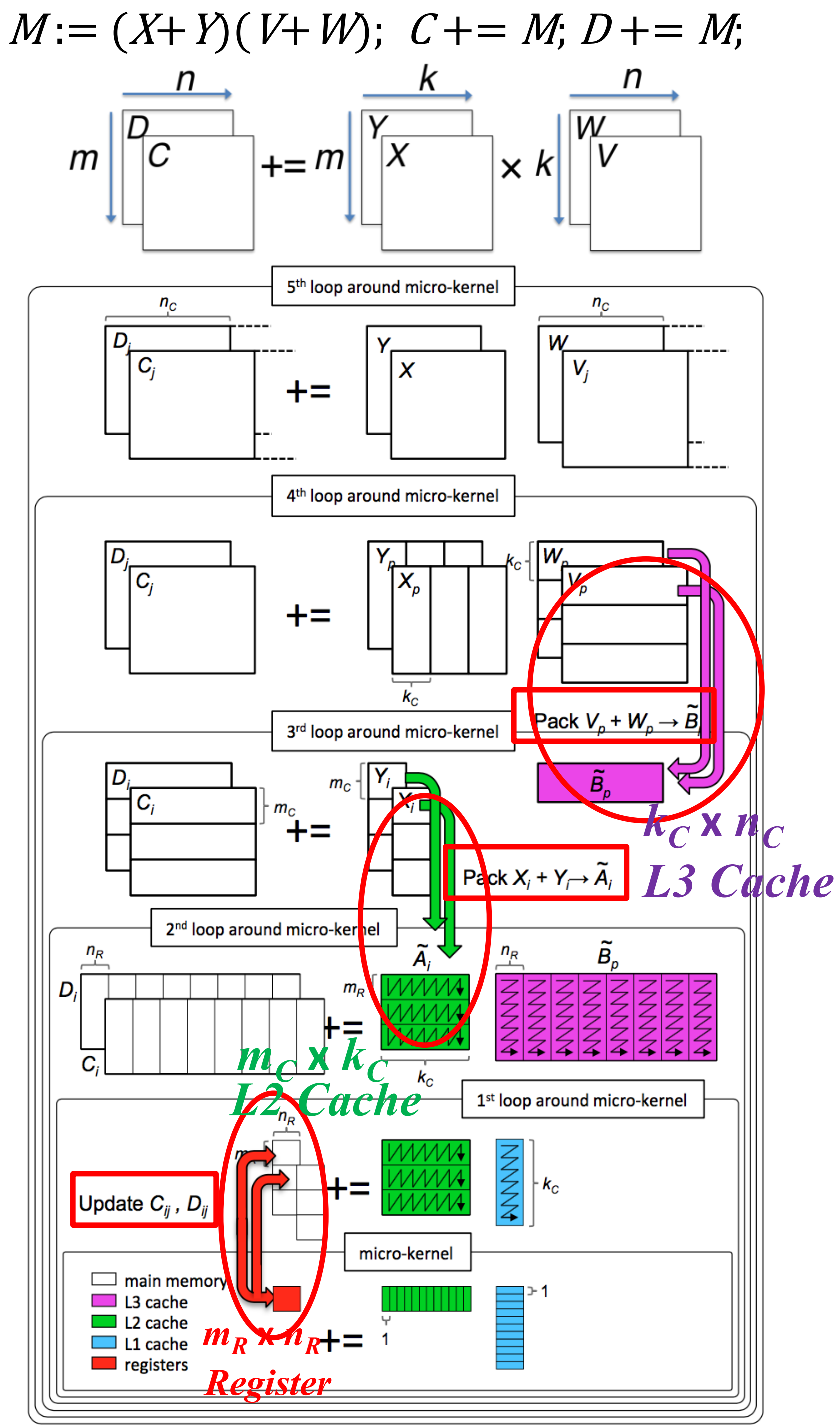
$$M_0 := (A_{00}+A_{11})(B_{00}+B_{11}); \quad C_{00} \mathrel{+}= M_0; C_{11} \mathrel{+}= M_0;$$

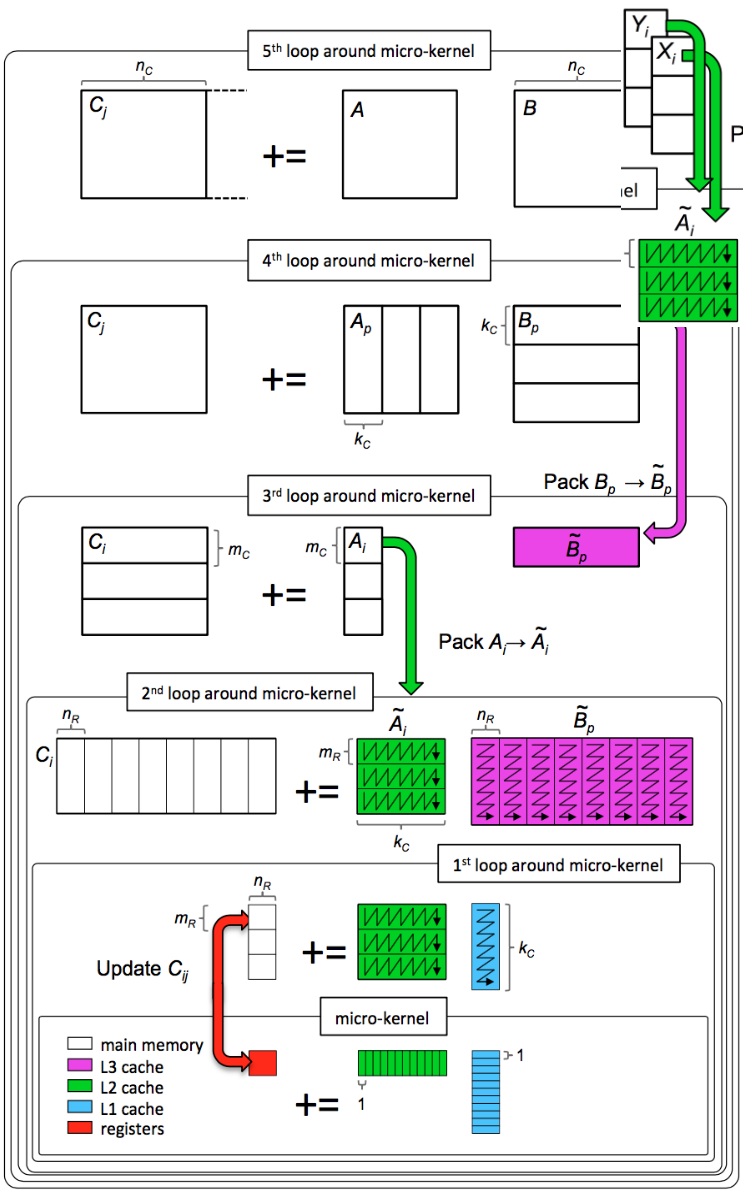
- Packing routines:
  - Implicit tensor-to-matrix permutations
  - Addition of submatrices of A and B.
- Micro-kernel:
  - Implicit matrix-to-tensor transformations
  - Scatter update of submatrices of C.

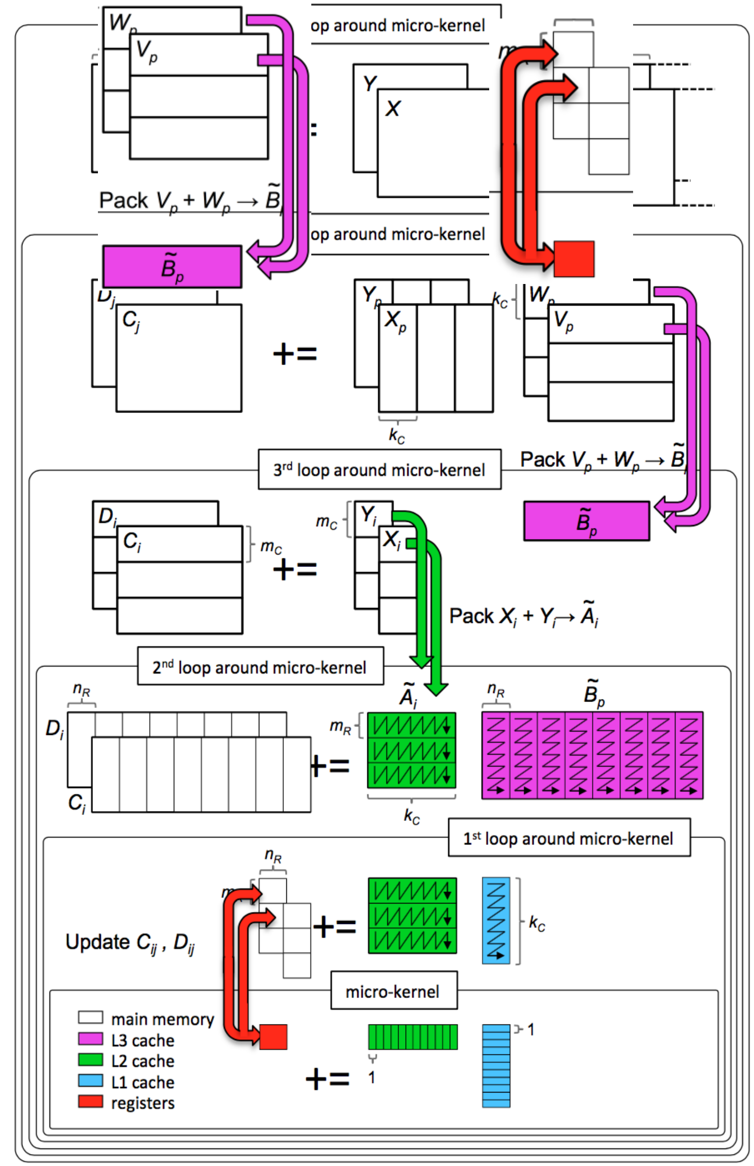
❑ ~~Additional workspace for Transposition (Tensor Contraction)~~

❑ ~~Additional Workspace for Summation (Strassen)~~

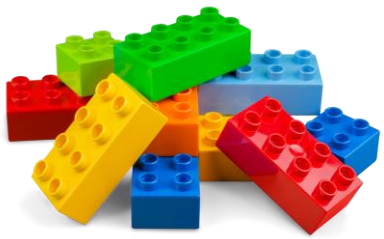

C += AB;
5th loop around micro-kernel
4th loop around micro-kernel
Pack $B_p \rightarrow \tilde{B}_p$
3rd loop around micro-kernel
Pack $A_i \rightarrow \tilde{A}_i$
2nd loop around micro-kernel
1st loop around micro-kernel
Update C_{ij}
micro-kernel
main memory
L3 cache
L2 cache
L1 cache
registers

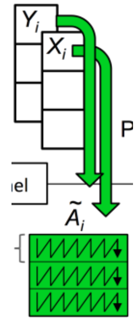

M := (X+Y)(V+W); C += M; D += M;
5th loop around micro-kernel
4th loop around micro-kernel
Pack $V_p + W_p \rightarrow \tilde{B}_p$
3rd loop around micro-kernel
Pack $X_i + Y_i \rightarrow \tilde{A}_i$
k_C x n_C
L3 Cache
2nd loop around micro-kernel
m_C x k_C
L2 Cache
1st loop around micro-kernel
Update C_{ij}, D_{ij}
micro-kernel
main memory
L3 cache
L2 cache
L1 cache
registers
m_R x n_R
Register

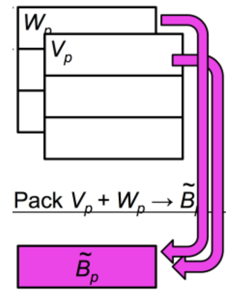

5th loop around micro-kernel
n_C
C_j
+=
A
B
Y_i
X_i
4th loop around micro-kernel
Pack B_p → B̃_p
3rd loop around micro-kernel
m_C
Pack A_i → Ã_i
2nd loop around micro-kernel
n_R
m_R
k_C
1st loop around micro-kernel
Update C_ij
micro-kernel
main memory
L3 cache
L2 cache
L1 cache
registers

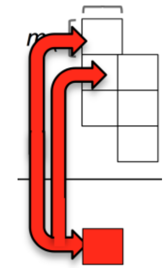

W_p
V_p
Pack V_p + W_p → B̃
Y
X
D_j
C_j
Y_p
X_p
k_C
Pack V_p + W_p → B̃
3rd loop around micro-kernel
D_i
C_i
m_C
Pack X_i + Y_i → Ã_i
2nd loop around micro-kernel
n_R
m_R
1st loop around micro-kernel
Update C_ij , D_ij
micro-kernel
main memory
L3 cache
L2 cache
L1 cache
registers

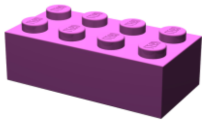
Variations on a theme











• Naïve Strassen	✗	✗	✗
• AB Strassen	✔	✔	✗
• ABC Strassen	✔	✔	✔

Outline

- Background
  - High-performance GEMM
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Performance Model

- Performance Metric

$$\boxed{\mathcal{C}_{\Pi_{\mathcal{C}}(I_m J_n)} := \sum_{P_k \in N_{p_0} \times \ldots \times N_{p_{k-1}}} \mathcal{A}_{\Pi_{\mathcal{A}}(I_m P_k)} \cdot \mathcal{B}_{\Pi_{\mathcal{B}}(P_k J_n)} + \mathcal{C}_{\Pi_{\mathcal{C}}(I_m J_n)}}$$

$$\begin{aligned} N_{I_m} &= \textstyle\prod_{i \in I_m} N_i = N_{i_0} \cdot \ldots \cdot N_{i_{m-1}}, \\ N_{J_n} &= \textstyle\prod_{j \in J_n} N_j = N_{j_0} \cdot \ldots \cdot N_{j_{n-1}}, \\ N_{P_k} &= \textstyle\prod_{p \in P_k} N_p = N_{p_0} \cdot \ldots \cdot N_{p_{k-1}}. \end{aligned}$$

$$\textit{Effective}\ \text{GFLOPS} = \frac{2 \cdot N_{I_m} \cdot N_{J_n} \cdot N_{P_k}}{\text{T (in seconds)}} \cdot 10^{-9}$$

- Total Time Breakdown

$$T = T_a + T_m$$

Arithmetic Operations

Memory Operations

$$T_a = W_a^{\times} \cdot T_a^{\times} + W_a^{\mathcal{A}+} \cdot T_a^{\mathcal{A}+} + W_a^{\mathcal{B}+} \cdot T_a^{\mathcal{B}+} + W_a^{\mathcal{C}+} \cdot T_a^{\mathcal{C}+}$$

	type	τ	TBLIS	L-level
$T_a^{\times}$	-	τ_a	$2N_{I_m}N_{J_n}N_{P_k}$	$2\frac{N_{I_m}}{2^L}\frac{N_{J_n}}{2^L}\frac{N_{P_k}}{2^L}$
$T_a^{\mathcal{A}+}$	-	τ_a	-	$2\frac{N_{I_m}}{2^L}\frac{N_{P_k}}{2^L}$
$T_a^{\mathcal{B}+}$	-	τ_a	-	$2\frac{N_{P_k}}{2^L}\frac{N_{J_n}}{2^L}$
$T_a^{\mathcal{C}+}$	-	τ_a	-	$2\frac{N_{I_m}}{2^L}\frac{N_{J_n}}{2^L}$

	TBLIS	1-level			2-level		
		ABC	AB	Naive	ABC	AB	Naive
$W_a^{\times}$	1	7	7	7	49	49	49
$W_a^{\mathcal{A}+}$	-	5	5	5	95	95	95
$W_a^{\mathcal{B}+}$	-	5	5	5	95	95	95
$W_a^{\mathcal{C}+}$	-	12	12	12	144	144	144

$$N_{I_m} = \prod_{i \in I_m} N_i = N_{i_0} \cdot \ldots \cdot N_{i_{m-1}},$$
$$N_{J_n} = \prod_{j \in J_n} N_j = N_{j_0} \cdot \ldots \cdot N_{j_{n-1}},$$
$$N_{P_k} = \prod_{p \in P_k} N_p = N_{p_0} \cdot \ldots \cdot N_{p_{k-1}}.$$

$$T_a = W_a^{\times} \cdot T_a^{\times} + W_a^{\mathcal{A}+} \cdot T_a^{\mathcal{A}+} + W_a^{\mathcal{B}+} \cdot T_a^{\mathcal{B}+} + W_a^{\mathcal{C}+} \cdot T_a^{\mathcal{C}+}$$
$$T_m = W_m^{\mathcal{A}\times} \cdot T_m^{\mathcal{A}\times} + W_m^{\mathcal{B}\times} \cdot T_m^{\mathcal{B}\times} + W_m^{\mathcal{C}\times} \cdot T_m^{\mathcal{C}\times} + W_m^{\mathcal{A}+} \cdot T_m^{\mathcal{A}+} + W_m^{\mathcal{B}+} \cdot T_m^{\mathcal{B}+} + W_m^{\mathcal{C}+} \cdot T_m^{\mathcal{C}+}$$

	type	τ	TBLIS	L-level
$T_a^{\times}$	-	τ_a	$2N_{I_m}N_{J_n}N_{P_k}$	$2\frac{N_{I_m}}{2^L}\frac{N_{J_n}}{2^L}\frac{N_{P_k}}{2^L}$
$T_a^{\mathcal{A}+}$	-	τ_a	-	$2\frac{N_{I_m}}{2^L}\frac{N_{P_k}}{2^L}$
$T_a^{\mathcal{B}+}$	-	τ_a	-	$2\frac{N_{P_k}}{2^L}\frac{N_{J_n}}{2^L}$
$T_a^{\mathcal{C}+}$	-	τ_a	-	$2\frac{N_{I_m}}{2^L}\frac{N_{J_n}}{2^L}$
$T_m^{\mathcal{A}\times}$	`r`	τ_b	$N_{I_m}N_{P_k}\lceil\frac{N_{J_n}}{n_c}\rceil$	$\frac{N_{I_m}}{2^L}\frac{N_{P_k}}{2^L}\lceil\frac{N_{J_n}/2^L}{n_c}\rceil$
$T_m^{\widetilde{A}\times}$	`w`	τ_b	$N_{I_m}N_{P_k}\lceil\frac{N_{J_n}}{n_c}\rceil$	$\frac{N_{I_m}}{2^L}\frac{N_{P_k}}{2^L}\lceil\frac{N_{J_n}/2^L}{n_c}\rceil$
$T_m^{\mathcal{B}\times}$	`r`	τ_b	$N_{J_n}N_{P_k}$	$\frac{N_{J_n}}{2^L}\frac{N_{P_k}}{2^L}$
$T_m^{\widetilde{B}\times}$	`w`	τ_b	$N_{J_n}N_{P_k}$	$\frac{N_{J_n}}{2^L}\frac{N_{P_k}}{2^L}$
$T_m^{\mathcal{C}\times}$	`r/w`	τ_b	$2\lambda N_{I_m}N_{J_n}\lceil\frac{N_{P_k}}{k_c}\rceil$	$2\lambda\frac{N_{I_m}}{2^L}\frac{N_{J_n}}{2^L}\lceil\frac{N_{P_k}/2^L}{k_c}\rceil$
$T_m^{\mathcal{A}+}$	`r/w`	τ_b	$N_{I_m}N_{P_k}$	$\frac{N_{I_m}}{2^L}\frac{N_{P_k}}{2^L}$
$T_m^{\mathcal{B}+}$	`r/w`	τ_b	$N_{J_n}N_{P_k}$	$\frac{N_{J_n}}{2^L}\frac{N_{P_k}}{2^L}$
$T_m^{\mathcal{C}+}$	`r/w`	τ_b	$N_{I_m}N_{J_n}$	$\frac{N_{I_m}}{2^L}\frac{N_{J_n}}{2^L}$

	TBLIS	1-level			2-level		
		ABC	AB	Naive	ABC	AB	Naive
$W_a^{\times}$	1	7	7	7	49	49	49
$W_a^{\mathcal{A}+}$	-	5	5	5	95	95	95
$W_a^{\mathcal{B}+}$	-	5	5	5	95	95	95
$W_a^{\mathcal{C}+}$	-	12	12	12	144	144	144
$W_m^{\mathcal{A}\times}$	1	12	12	7	194	194	49
$W_m^{\widetilde{A}\times}$	-	-	-	-	-	-	-
$W_m^{\mathcal{B}\times}$	1	12	12	7	194	194	49
$W_m^{\widetilde{B}\times}$	-	-	-	-	-	-	-
$W_m^{\mathcal{C}\times}$	1	12	7	7	144	49	49
$W_m^{\mathcal{A}+}$	-	-	-	19	-	-	293
$W_m^{\mathcal{B}+}$	-	-	-	19	-	-	293
$W_m^{\mathcal{C}+}$	-	-	36	36	-	432	432

$$N_{I_m} = \prod_{i\in I_m} N_i = N_{i_0} \cdot \ldots \cdot N_{i_{m-1}},$$
$$N_{J_n} = \prod_{j\in J_n} N_j = N_{j_0} \cdot \ldots \cdot N_{j_{n-1}},$$
$$N_{P_k} = \prod_{p\in P_k} N_p = N_{p_0} \cdot \ldots \cdot N_{p_{k-1}}.$$

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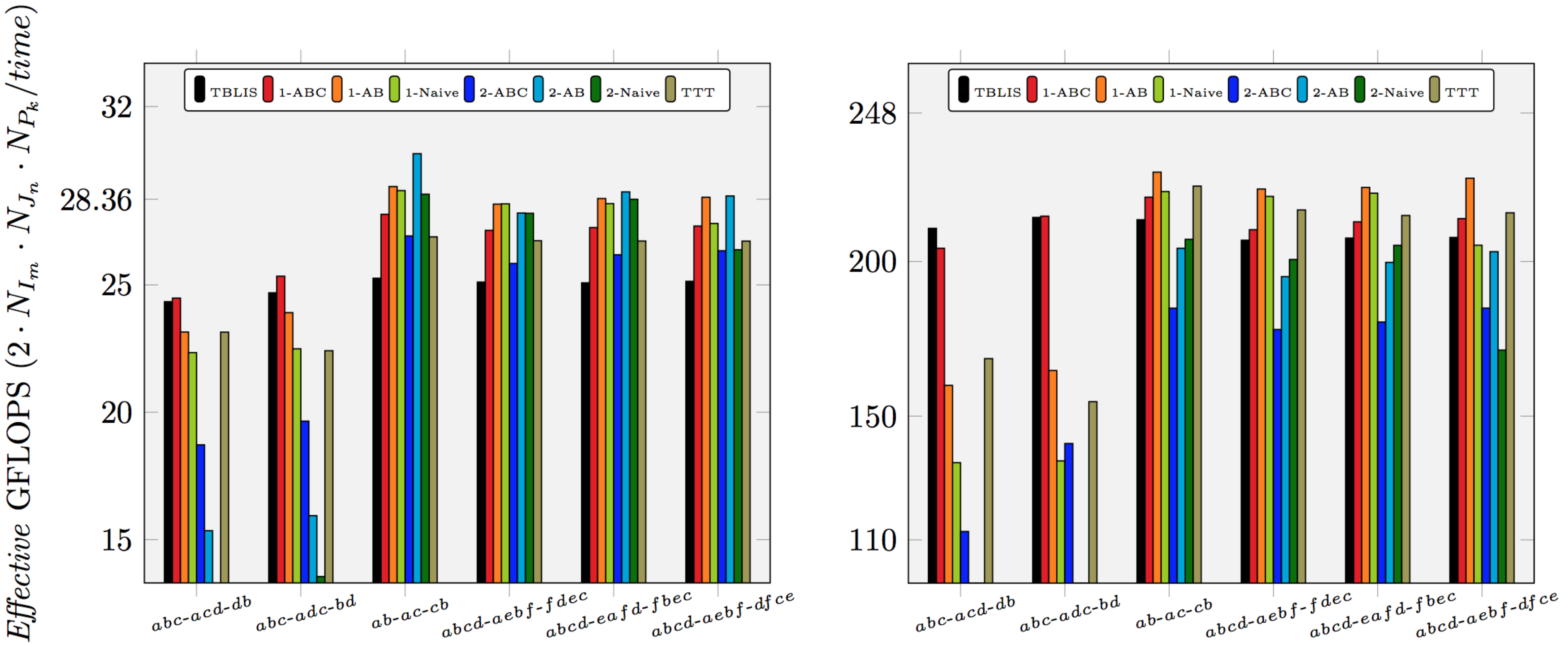
Real-world Benchmark

Intel Xeon E5-2680 v2 (Ivy Bridge, 10 core/socket)



$\mathcal{C}_{abcd} += \mathcal{A}_{aebf}\mathcal{B}_{dfce}$ is denoted as $abcd\text{-}aebf\text{-}dfce$

Paul Springer, and Paolo Bientinesi. "Design of a high-performance GEMM-like tensor-tensor multiplication." arXiv preprint arXiv:1607.00145 (2016).

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Conclusion

- First work to leverage Strassen's Algorithm for Tensor Contraction.
- Fusing matrix summation and permutations with packing and micro-kernel operations inside GEMM.
- Avoiding explicit transpositions and extra workspace, and reducing the overhead of memory movement.
- Achieving ~1.3x speedup on single core and multicore parallel architectures.

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Thank you!

The source code can be downloaded from:
https://github.com/flame/tblis-strassen